

C4 PARTIAL FRACTIONS

Worksheet A

1 Find the values of the constants A and B in each identity.

a $x - 8 \equiv A(x - 2) + B(x + 4)$

b $6x + 7 \equiv A(2x - 1) + B(x + 2)$

2 Find the values of the constants A and B in each identity.

a $\frac{2}{(x+1)(x+3)} \equiv \frac{A}{x+1} + \frac{B}{x+3}$

b $\frac{x-3}{x(x-1)} \equiv \frac{A}{x} + \frac{B}{x-1}$

c $\frac{x+1}{(x-3)(x-5)} \equiv \frac{A}{x-3} + \frac{B}{x-5}$

d $\frac{x+10}{(1+x)(2-x)} \equiv \frac{A}{1+x} + \frac{B}{2-x}$

e $\frac{4x-1}{x^2+x-2} \equiv \frac{A}{x+2} + \frac{B}{x-1}$

f $\frac{x-9}{x^2-4x+3} \equiv \frac{A}{x-1} + \frac{B}{x-3}$

3 Express in partial fractions

a $\frac{8}{(x-1)(x+3)}$

b $\frac{x-1}{(x+2)(x+3)}$

c $\frac{10x}{(x+4)(x-1)}$

d $\frac{5x+7}{x^2+x}$

e $\frac{x+2}{x^2-5x+4}$

f $\frac{4x+6}{x^2-9}$

g $\frac{3x+2}{x^2-2x-24}$

h $\frac{38-x}{12-x-x^2}$

i $\frac{4x-5}{(2x+1)(x-3)}$

j $\frac{1-3x}{(3x+4)(2x+1)}$

k $\frac{x+1}{x-3x^2}$

l $\frac{5}{2x^2+3x-2}$

m $\frac{2(x+5)}{8x^2+10x-3}$

n $\frac{3x-7}{x^2-2x-3}$

o $\frac{1-3x}{1-x-2x^2}$

4 Find the values of the constants A , B and C in each identity.

a $3x^2 + 17x - 32 \equiv A(x - 1)(x + 3) + B(x - 1)(x - 4) + C(x + 3)(x - 4)$

b $14x + 2 \equiv A(x + 1)(x - 2) + B(x + 1)(3x - 1) + C(x - 2)(3x - 1)$

c $x^2 + x + 12 \equiv A(x + 1)^2 + B(x + 1)(x + 5) + C(x + 5)$

d $4(5x^2 + 4) \equiv A(2x + 1)^2 + B(2x + 1)(x - 3) + C(x - 3)$

5 Find the values of the constants A , B and C in each identity.

a $\frac{8x+14}{(x-2)(x+1)(x+3)} \equiv \frac{A}{x-2} + \frac{B}{x+1} + \frac{C}{x+3}$

b $\frac{2x^2-6x+20}{(x+1)(x+2)(x-6)} \equiv \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{x-6}$

c $\frac{9x-14}{(x+4)(x-1)^2} \equiv \frac{A}{x+4} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$

d $\frac{3x^2-7x-4}{(x-3)(x-2)^2} \equiv \frac{A}{x-3} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$

C4 PARTIAL FRACTIONS

Worksheet A continued

6 Express in partial fractions

$$\text{a } \frac{2x^2 + 4}{x(x-1)(x-4)}$$

$$\text{b } \frac{9}{(x-2)(x+1)^2}$$

$$\text{c } \frac{x^2 + 11x - 21}{(2x+1)(x-2)(x-3)}$$

$$\text{d } \frac{10x+9}{(x-4)(x+3)^2}$$

$$\text{e } \frac{x^2 + 4x + 5}{(x+1)(x+2)^2}$$

$$\text{f } \frac{16-2x}{(x-3)(x^2-4)}$$

$$\text{g } \frac{2-9x}{(x-3)(2x-1)^2}$$

$$\text{h } \frac{3+24x-4x^2}{(x+1)(x-4)^2}$$

$$\text{i } \frac{9x^2 - 2x - 12}{x^3 + x^2 - 6x}$$

$$\text{j } \frac{5x^2 + 3x - 20}{x^3 + 4x^2}$$

$$\text{k } \frac{13-3x^2}{(2x+3)(x-1)^2}$$

$$\text{l } \frac{26-x-x^2}{(x-1)(x+3)(x+5)}$$

7 Find the values of the constants A , B and C in each identity.

$$\text{a } \frac{x^2}{(x-2)(x-6)} \equiv A + \frac{B}{x-2} + \frac{C}{x-6}$$

$$\text{b } \frac{x^2 + 2x + 9}{x^2 + 4x - 5} \equiv A + \frac{B}{x-1} + \frac{C}{x+5}$$

8 a Find the quotient and remainder obtained in dividing $(x^3 + 4x^2 - 2)$ by $(x^2 + x - 2)$.

b Hence, express $\frac{x^3 + 4x^2 - 2}{x^2 + x - 2}$ in partial fractions.

9 Express in partial fractions

$$\text{a } \frac{x^2 + 3}{(x-3)(x+1)}$$

$$\text{b } \frac{x^3 - 3x^2 - x + 2}{x^2 - 4}$$

$$\text{c } \frac{2x^2 + 7x}{x^2 + 6x + 8}$$

$$\text{d } \frac{3(x+1)(x-1)}{(x-4)(x+5)}$$

$$\text{e } \frac{3x^3 + 7x^2 + 4}{x^2 + 4x + 3}$$

$$\text{f } \frac{4x^2 - 7x + 5}{2x^2 - 7x + 3}$$

$$\text{g } \frac{2x^2}{x^2 - 2x - 3}$$

$$\text{h } \frac{x^3 - 6x^2 + 6x + 1}{x^2 - 6x + 5}$$

$$\text{i } \frac{9x^3 - 27x - 2}{3x^2 - 4x - 4}$$

$$10 \quad f(x) = \frac{x+5}{(x-1)(2x+1)}$$

a Express $f(x)$ in partial fractions.

b Find the exact x -coordinates of the stationary points of the curve $y = f(x)$.

$$11 \quad f(x) = \frac{x(4x+5)}{(x-1)(x+2)^2}$$

a Find the values of the constants A , B and C such that

$$f(x) = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$$

b Show that the tangent to the curve $y = f(x)$ at the point where $x = -1$ has the equation

$$3x - 4y + 5 = 0.$$